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## Effect of Diurnal Convection on Trapped Thermal Plasma in the Outer Plasmasphere

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# **Effect of diurnal convection on trapped thermal plasma in the outer plasmasphere**

## **1. Introduction**

The effects of magnetospheric convection, and its associated large-scale electric field, on the global structure and morphology of the magnetosphere are well known [Axford, 1969]. The most striking example in the inner magnetosphere is the formation of the plasmapause, whose location is determined by the interplay between the large-scale electric field and the corotation electric field [Nishida, 1966; Lemaire, 1974]. The effects of convection on the detailed structure of the region inside the plasmapause are less well known.

Recent work on plasmaspheric convection has utilized fluid methods [Rasmussen and Schunk, 1990; Rasmussen *et al.*, 1993; Khazanov *et al.*, 1994], which treat the entire distribution of particles identically. This is reasonable if one assumes that the flux tubes are completely filled, i.e., a diffusive equilibrium condition exists. However, convection (the diurnal motion of a magnetic flux tube) affects different regions of the plasma phase space in fundamentally different ways, leading to a steady state in the high-altitude regions near the plasmapause, but not hydrodynamic or thermal equilibrium. Because this partition in phase space is not realizable in a fluid formulation, it is necessary to treat the plasma from a kinetic point of view, particularly at high altitudes where Coulomb collisions are infrequent. We also note that differences between measured densities and fluid model predictions have been previously reported [Rasmussen and Schunk, 1990; Craven *et al.*, 1997].

In this Letter, we use collisionless kinetic theory to investigate the convection of magnetically trapped thermal particles, and to elucidate their role in determining the density and temperature morphology inside the plasmapause. This trapped population, found in high-altitude flux tubes, is thermodynamically isolated from the ionospheric plasma on transport time scales, and is strongly affected by the convection electric fields which form the plasma-

pause. These flux tubes experience expansion and contraction due to radial excursions during their diurnal trajectory. This affects the phase-space density of the trapped population. Flux-tube motion does not, however, modify the untrapped population because those particles escape and re-enter the ionosphere on a time scale ( $\sim 1$  hour) [Lemaire, 1989] shorter than that of convection ( $\sim 12$  hours).

In addition, the present approach has the potential for investigating a wide variety of instabilities (due to the nonequilibrium nature of the distribution function) with a possibility of explaining the observed asymmetric wave activity in the outer plasmasphere [Boardsen *et al.*, 1995]. The waves could provide a mechanism for scattering particles between the trapped population and the untrapped population. These velocity-space diffusion effects are not included in the present analysis.

## 2. Convection model

We divide the plasmasphere into two regions: the barosphere, where Coulomb collisions dominate and the plasma distribution function is Maxwellian; and the region above the barosphere, which is taken to be collisionless, and where the convection of trapped particles is included. The boundary between these two regions is the baropause, approximated by an infinitesimally thin surface. Beyond the plasmapause, the trapped particles are absent because they are continually convected away toward the magnetopause.

For this study, we consider the low-energy limit, ignoring both curvature and gradient drifts: all particle motion is field aligned, with both energy and magnetic moment conserved. In this limit, there are four classes of particles possible in a closed-field-line region, and they can be identified by their position in velocity space [Lemaire and Scherer, 1970]: (1) escaping particles which have small pitch angles, enough energy to escape the gravitational trap, and can enter the other hemisphere, (2) incoming particles which escape from one hemisphere and enter the other, (3) ballistic particles which do not have enough energy to overcome gravity and return to the ionosphere, and (4) trapped particles which are reflected between two mirror points and

never encounter the baropause. The first three classes are in contact with the ionosphere and together make up the portion of the distribution called the source cone. Because of this thermal contact, the characteristics of the source-cone particles, such as temperature and density, are determined by ionospheric conditions. The fourth class (trapped particles) forms a loss cone in velocity space, and can exhibit a morphology significantly different from the ionospherically generated source cone, even during magnetically quiet times.

The velocity distribution is assumed to be Maxwellian at the baropause, transforms into a source-cone distribution as it flows up the field line, and re-enters the barosphere. The trapped particles in each flux tube, however, remain in the flux tube indefinitely (in the limit of no diffusion), and convect under the influence of the combined convection and corotation electric fields. Only those trapped particles which never encounter the baropause during their diurnal motion (and hence would be absorbed) are included. Also, only those trapped particles which drift entirely around the Earth are included. The distribution of the trapped particles is taken to be Maxwellian over the permitted region of phase space, and their density, relative to that of the source cone, is a free parameter. We do not, however, specify a mechanism for filling the loss cone.

To determine the source-cone distribution quantitatively, the lower boundary condition of a given field line is a normalized, isotropic Maxwellian distribution  $f_0$  (at the baropause) for each species with mass  $m$

$$f_0(\mathbf{v}_0) = n_0 \left( \frac{m}{2\pi k T_0} \right)^{3/2} \exp \left\{ -\frac{m(v_{||0}^2 + v_{\perp0}^2)}{2kT_0} \right\}, \quad (1)$$

where  $n_0$ ,  $T_0$ , and  $\mathbf{v}_0$  are the baropause density, temperature, and velocity, respectively. The source-cone distribution  $f_s$  at a higher altitude  $\mathbf{r}$  on the same field line is determined by applying Liouville's theorem (conserving energy and magnetic moment) and taking into account the accessibility condition

$$\begin{aligned} f_s(\mathbf{v}, \mathbf{r}) &= f_0(\mathbf{v}_0) \\ &\times \Theta \left( \frac{B}{B_0 - B} \left[ v_{||}^2 + 2 \frac{U - U_0}{m} \right] - v_{\perp}^2 \right), \end{aligned} \quad (2)$$

where  $\Theta$  is the unit step function,  $B$  and  $U$  are the magnetic field and the potential energy at  $\mathbf{r}$ ,  $B_0$  and  $U_0$  are those quantities at the baropause, and  $\mathbf{v}_0(\mathbf{v})$  is the velocity transformation (see, e.g., *Eviatar et al.* [1964]). The potential energy  $U$  consists of the gravitational, electrostatic and centrifugal potential energies. The  $\Theta$  function in Eq. (2) indicates the region of phase space that the source cone occupies. The density of the source-cone population is determined by integrating the distribution function,  $n_s = \int d^3v f_s$ , and one obtains

$$n_s(\mathbf{r}) = n_0 \left( e^{-\psi} - \sqrt{A} e^{-\psi/A} \right), \quad (3)$$

where  $\psi = (U - U_0)/kT_0$  is the dimensionless potential energy and  $A = 1 - B/B_0$ . This result is equivalent to that obtained by *Eviatar et al.* [1964]. The first term in Eq. (3) corresponds to isothermal diffusive equilibrium, while the trapped particles are subtracted by the second term.

To determine the trapped distribution quantitatively, we include the effects of convection on the region of phase space left undetermined by Eq. (2). We assume that the distribution in this region is proportional to  $f_0$ , but that accessibility limits the trapped distribution  $f_t$  to

$$\begin{aligned} f_t(\mathbf{v}, \mathbf{r}) &= \eta f_0(\mathbf{v}_0) \\ &\times \Theta \left( v_\perp^2 - \frac{B}{B_{0d} - B} \left[ v_\parallel^2 + 2 \frac{U - U_{0d}}{m} \right] \right) \\ &\times \Theta \left( \frac{B}{B_d - B} \left[ v_\parallel^2 + 2 \frac{U - U_d}{m} \right] - v_\perp^2 \right), \end{aligned} \quad (4)$$

where  $\eta$  is a free parameter which characterizes the density of the trapped population relative to the source cone,  $B_d$  and  $U_d$  are the magnetic field and potential energy at the equator of the flux tube at its closest to Earth (which we take to be at dawn—this is true for a symmetric electric field), and  $B_{0d}$  and  $U_{0d}$  are the same quantities at the baropause of the dawn flux tube. Those parameters with a subscript  $d$  are controlled by the electric field model, which determines the drift path of the flux tube. The parameter  $\eta$  will depend on flux-tube filling and loss processes, as well as the recent history of the plasmasphere. The first  $\Theta$  function in Eq. (4) allows only those particles that do not encounter the baropause during their diurnal convection. That is, we keep only those trapped particles whose turning point is always above the baropause. The second

$\Theta$  function in Eq. (4) is the accessibility condition from dawn for convective motion. That is, we keep only those particles which actually drift completely around the Earth. Equation (4) is the distribution only in the equatorial plane; a similar expression holds at higher latitudes. Integrating over the distribution, the density of the drifting trapped particles,  $n_t$ , is

$$n_t(\mathbf{r}) = \eta n_0 \left( \sqrt{A_{0d}} e^{-\psi/A_{0d}} - \sqrt{A_d} e^{-\psi-(\psi_d/\beta)} \right), \quad (5)$$

where  $A_{0d} = 1 - B/B_{0d}$ ,  $A_d = 1 - B/B_d$ ,  $\psi_d = (U - U_d)/kT_0$ , and  $\beta = A_d/(1 - A_d)$ . Figure 1 shows the regions of velocity space which the two distinct populations occupy, at the equator of a typical flux tube.

The total density is the sum of the two populations,  $n = n_s + n_t$ , as given by Eqs. (3) and (5). In these expressions the electrostatic potential is undetermined and must be calculated self-consistently by applying the quasineutrality condition  $n_e = \sum_i n_i$ .

Besides the density, knowledge of the total distribution allows the calculation of the effective temperature. The distribution functions calculated above exhibit highly anisotropic temperatures, which could lead to strong instabilities. The actual spatial regions of instability will depend on the parameter  $\eta$ , as well as the specific electric field model.

### 3. Equatorial densities

To illustrate the result of treating the trapped particles in the manner described, we now calculate plasma densities in the equatorial plane using the plasmapause formation mechanism of *Nishida* [1966] (corotation and convection electric fields combine to form a separatrix), a uniform dawn-dusk electric field [*Kavanagh et al.*, 1968], a dipolar magnetic field aligned with the rotation axis, a baropause of constant height, density and temperature, with only the gravitational and electrostatic potential energies included in  $U$  (no centrifugal potential). These simplifications allow us to focus on the contribution of the trapped population, and they can easily be relaxed. For illustrative purposes, we choose two ion species, 90% hydrogen and 10% helium at the baropause, along with  $r_0 = 3000$  km,  $T_0 = 3000$  K, and a dawn-dusk electric

field of 0.25 mV/m. A detailed study of parameter space is beyond the scope of this Letter.

Figure 2 shows the electron density in the equatorial plane for  $\eta = 1.75$ . Inside the plasmapause (shown with the characteristic teardrop shape expected from the electric field model), the density enhancement in the dawn sector and the density depletion in the dusk sector are due primarily to the convection of the trapped population. Outside the plasmapause the density is low because only source-cone particles are present.

Because of the importance of future helium imaging to our understanding of the plasma-sphere, Fig. 3 shows the ratio of helium ion density to hydrogen ion density in the equatorial plane. This ratio varies rapidly for  $r \lesssim 2R_E$ , but varies slowly for  $r \gtrsim 2R_E$ , in reasonable agreement with statistical studies [Craven *et al.*, 1997]. This behavior is due to the mass ratio of the two species and the interplay between the gravitational and electrostatic potentials. Figure 3 also shows that this ratio is approximately constant across the plasmapause, an observation noted previously [Horwitz *et al.*, 1986].

In order to assess the capability of imaging to distinguish between models, Fig. 4 shows the ratio of the helium ion density given by the present model to what the helium ion density would be if diffusive equilibrium conditions existed. The solid line indicates the surface where the ratio is unity. Downward of this surface, the loss cone is overfilled relative to the source cone; duskward of this surface (and outside the plasmapause), the loss cone is underfilled. The strong dawn-dusk asymmetry of this ratio affirms the distinguishability of the present model. To illustrate the effect of varying  $\eta$ , the dashed and dotted lines in Fig. 4 indicate the positions of the unity surfaces for the cases where  $\eta = 1.5$  and  $\eta = 2.0$ , respectively. (The densities for these two cases are not shown, but they exhibit a strong asymmetry as well.)

A strength of the present method is that it is not limited to a particular electric field specification or plasmapause formation mechanism; other electric field models [e.g., Volland, 1973] or plasmapause models [e.g., Lemaire, 1974] can be used. In addition, it is possible to incorporate the effects of the penetration of the polar convection electric field to low latitudes [Spiro *et al.*, 1988; Doe *et al.*, 1992].

## 4. Conclusion

We have shown that trapped particles drifting along diurnal convection equipotentials and adjusting their density self-consistently leads to a morphology of the outer plasmasphere which is significantly different from that obtained using the assumption of diffusive equilibrium. The density exhibits a strong asymmetry between the dawn and dusk sectors compared with a diffusive equilibrium assumption. The partition in phase space between trapped particles and untrapped particles, and the different effects of convection on the different classes of particles, requires a kinetic theory.

Future spacecraft missions will have the capability to globally image the inner magnetosphere (e.g., through scattering of the solar 30.4 nm line by  $\text{He}^+$ ), and provide significant progress in our understanding of the physical processes in the plasmasphere [Meier, 1991; Williams *et al.*, 1992]. The present model has a predictive capability [Reynolds *et al.*, 1997] which is needed to extract quantitative information from the observational data.

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## References

- Axford, W. I., Magnetospheric convection, *Rev. Geophys.*, 7, 421, 1969.
- Boardsen, S. A., J. L. Green, S. F. Fung, and W. W. L. Taylor, Plasma wave imaging of the Earth's magnetosphere, *Eos Trans. AGU*, 76(46), Fall Meet. Suppl., F521, 1995.
- Craven, P. D., D. L. Gallagher, and R. H. Comfort, Relative concentration of He<sup>+</sup> in the inner magnetosphere as observed by the DE 1 retarding ion mass spectrometer, *J. Geophys. Res.*, 102, 2279, 1997.
- Doe, R. A., M. B. Moldwin, and M. Mendillo, Plasmapause morphology determined from an empirical ionospheric convection model, *J. Geophys. Res.*, 97, 1151, 1992.
- Eviatar, A., A. M. Lenchek, and S. F. Singer, Distribution of density in an ion-exosphere of a nonrotating planet, *Phys. Fluids*, 7, 1775, 1964.
- Horwitz, J. L., L. H. Brace, R. H. Comfort, and C. R. Chappell, Dual-spacecraft measurements of plasmasphere-ionosphere coupling, *J. Geophys. Res.*, 91, 11,203, 1986.
- Kavanagh, L. D., Jr., J. W. Freeman, Jr., and A. J. Chen, Plasma flow in the magnetosphere, *J. Geophys. Res.*, 73, 5511, 1968.
- Khazanov, G. V., C. E. Rasmussen, Yu. V. Konikov, T. I. Gombosi, and A. F. Nagy, Effect of magnetospheric convection on thermal plasma in the inner magnetosphere, *J. Geophys. Res.*, 99, 5923, 1994.
- Lemaire, J., The 'Roche-limit' of ionospheric plasma and the formation of the plasmapause, *Planet. Space Sci.*, 22, 757, 1974.
- Lemaire, J., Plasma distribution models in a rotating magnetic dipole and refilling of plasmaspheric flux tubes, *Phys. Fluids B*, 1, 1519, 1989.
- Lemaire, J., and M. Scherer, Model of the polar ion-exosphere, *Planet. Space Sci.*, 18, 103, 1970.
- Meier, R. R., Ultraviolet spectroscopy and remote sensing of the upper atmosphere, *Space Sci. Rev.*, 58, 1, 1991.

- Nishida, A., Formation of plasmapause, or magnetospheric plasma knee, by the combined action of magnetospheric convection and plasma escape from the tail, *J. Geophys. Res.*, 71, 5669, 1966.
- Rasmussen, C. E., and R. W. Schunk, A three-dimensional time-dependent model of the plasmasphere, *J. Geophys. Res.*, 95, 6133, 1990.
- Rasmussen, C. E., S. M. Guiter, and S. G. Thomas, A two-dimensional model of the plasmasphere: refilling time constants, *Planet. Space Sci.*, 41, 35, 1993.
- Reynolds, M. A., G. Ganguli, A. C. Nicholas, and R. R. Meier, Global images of plasmaspheric helium, *Eos Trans. AGU*, 78, Spring Meet. Suppl., 1997.
- Spiro, R. W., R. A. Wolf, and B. G. Fejer, Penetration of high-latitude-electric-field effects to low latitudes during SUNDIAL 1984, *Ann. Geophys.*, 6, 39, 1988.
- Volland, H., A semiempirical model of large-scale magnetospheric electric fields, *J. Geophys. Res.*, 78, 171, 1973.
- Williams, D. J., E. C. Roelof, and D. G. Mitchell, Global magnetospheric imaging, *Rev. Geophys.*, 30, 183, 1992.

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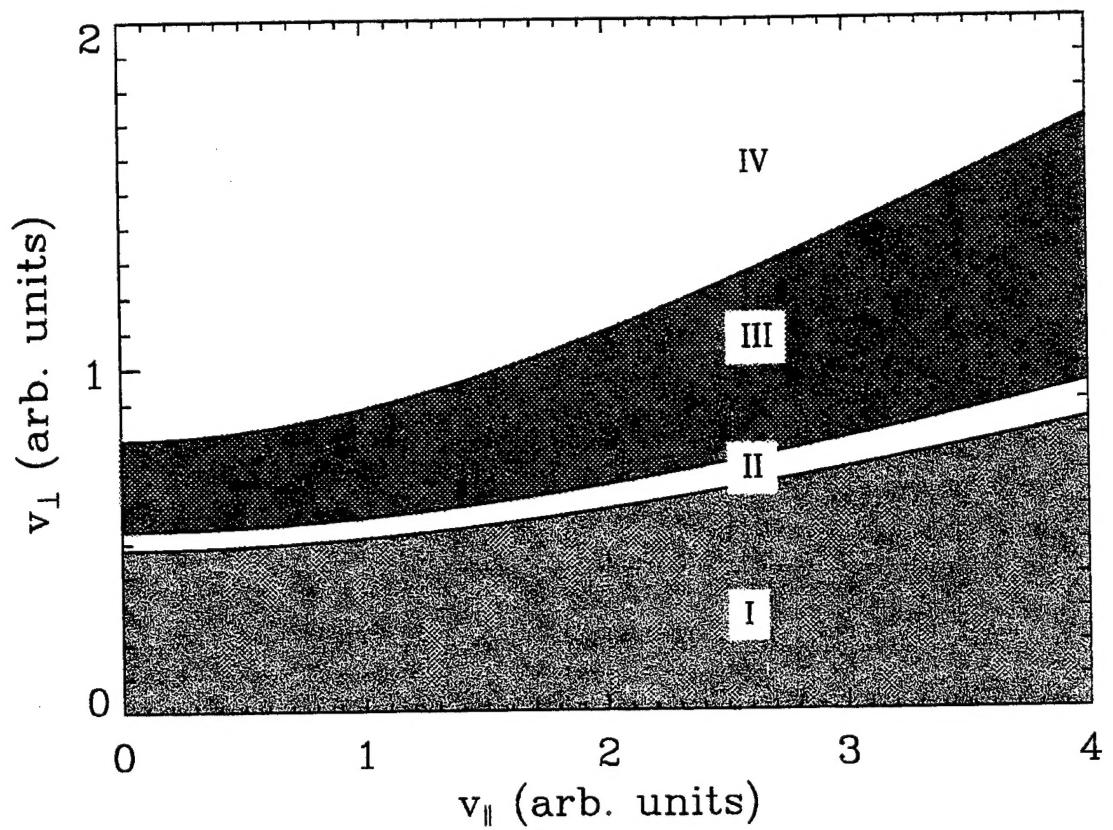
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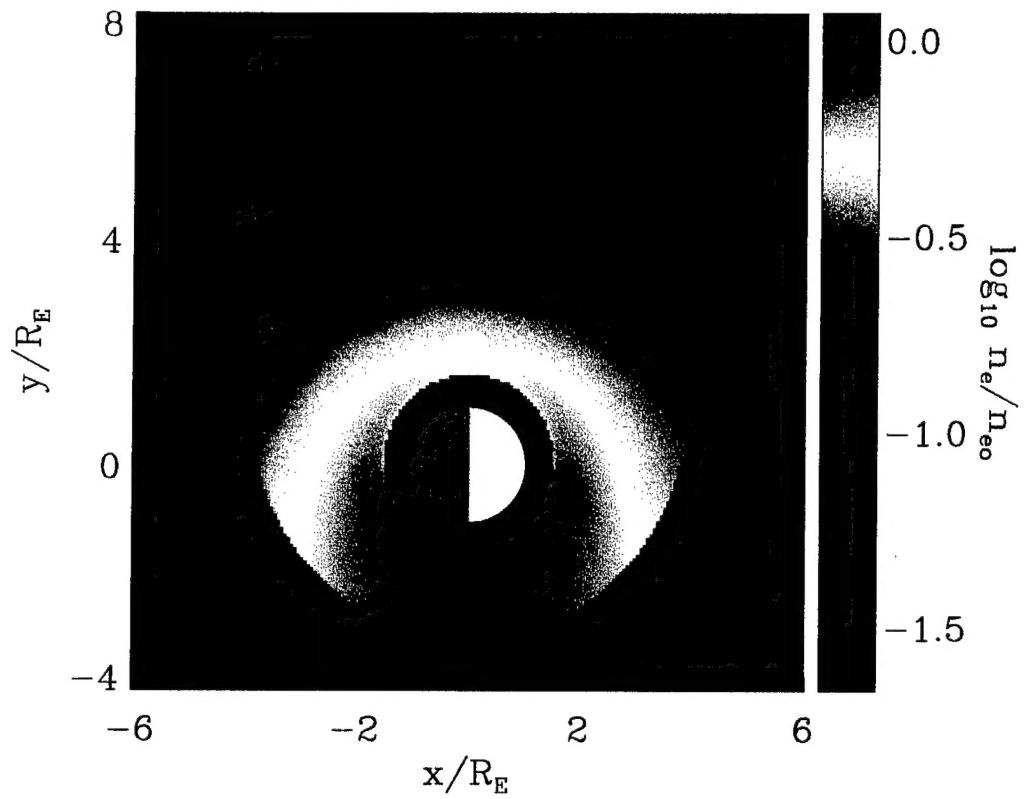
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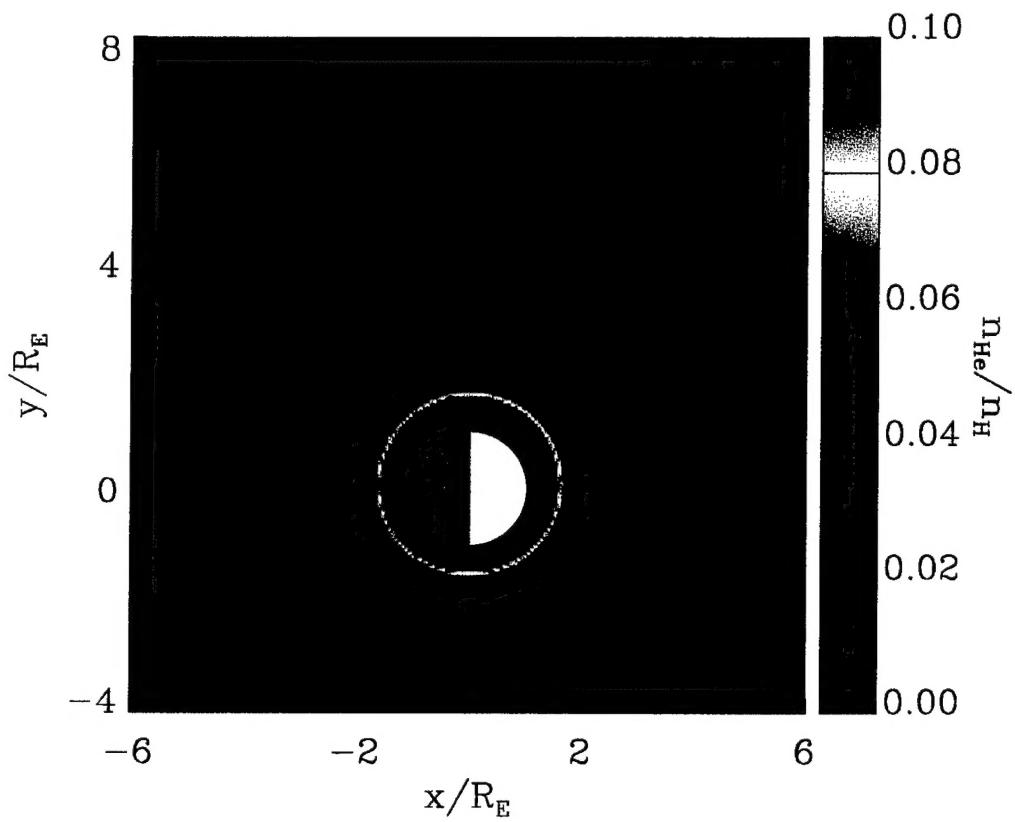
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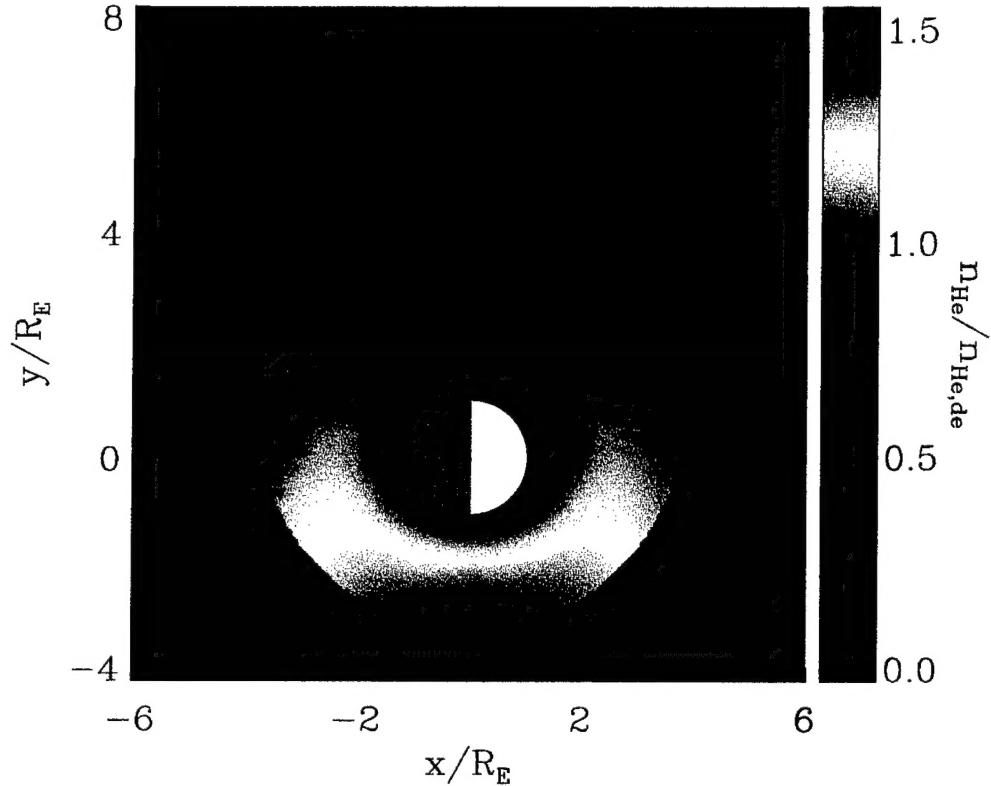
**Figure 1.** Velocity space at the equator of a typical flux tube. Region I is the source cone, and region III is the trapped population. Particles in region II encounter the baropause, and region IV is not accessible from dawn due to convection. The three boundaries between the regions are determined quantitatively by the three  $\Theta$  functions in Eqs. (2) and (4).



**Figure 2.** Electron density  $n_e$  in the equatorial plane, scaled to the electron density at the baropause,  $n_{eo}$ , using the GSM coordinate system. The parameters are  $\eta = 1.75$ ,  $r_0 = 3000$  km,  $T_0 = 3000$  K, 90% hydrogen and 10% helium at the baropause, and a dawn-dusk electric field of 0.25 mV/m.



**Figure 3.** Ratio of helium ion density  $n_{He}$  to hydrogen ion density  $n_H$  in the equatorial plane, for the same parameters as Fig. 2.



**Figure 4.** Ratio of helium ion density  $n_{He}$  to diffusive equilibrium helium ion density  $n_{He,de}$  in the equatorial plane, for the same parameters as Fig. 2. The solid line indicates the surface on which this ratio is unity. Near the plasmapause, this ratio is  $\sim 1.5$  at dusk and is  $\sim 0.25$  at dawn, a factor of 6 difference. The dashed and dotted lines indicate the positions of the unity surfaces for the cases where  $\eta = 1.5$  and  $\eta = 2.0$ , respectively. (The densities for these two cases are not shown.)